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Technical Notes

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Surrogate Modeling for Optimization of Dimpled Channel to Enhance Heat Transfer Performance

Abdus Samad,* Dong-Yoon Shin,* and Kwang-Yong Kim†
Inha University, Incheon 402-751, Republic of Korea
and
Tushar Goel‡ and Raphael T. Haftka§
University of Florida, Gainesville, Florida 32611

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I. Introduction

DIMPLES on solid surfaces usually used in cooling passages of a turbine blade along with ribs, pin fins, etc., as heat transfer augmentation devices, prevent the development of the thermal boundary layer and increase production of turbulent kinetic energy, and thus enhance turbulent heat transfer as reviewed by Ligrani et al. [1]. Inevitably, the surface deformations increase pressure loss in flow passage. The shape optimization of the dimpled surface, thus, is indispensable in compromising between enhancement of heat transfer and reduction of pressure loss. Kim and Choi [2] performed shape optimization of the dimpled channel to enhance turbulent heat transfer using the response surface method and they showed that the heat transfer rate increases by shape optimization. Optimizations of the rib-roughened channel using the response surface approximation (RSA) were performed by Kim and Kim [3] and Kim and Lee [4].

The surrogate models being widely used in multidisciplinary optimizations should be evaluated in two important aspects: computational economy that requires as few data points as possible for constructing a surrogate model, and accuracy in representing the characteristics of the design space. Queipo et al. [5] and Li and Padula [6] reviewed various surrogate-based models used in aerospace applications. Shyy et al. [7] applied global optimization methods to the rocket engine design. Zerpa et al. [8] presented a weighted average surrogate model using a local weighted average surrogate model based on RSA, radial basis neural network (RBNN), and Kriging (KRG) models. They determined weights for the weighted average model considering pointwise estimation of variance predicted by the three surrogate models. Goel et al. [9] developed a weighted surrogate model, using RSA, KRG, and RBNN models. The weights are determined according to cross-validation (CV) errors. The larger the error in the prediction of any surrogate, the lesser the weight assigned to the surrogate constructing the weighted average surrogate. They concluded that the weighted average surrogate model is a much more reliable prediction method than individual surrogates. Goel et al. [10] applied a surrogate model-based strategy for cryogenic cavitation model validation. Samad et al. [11] reported on the performances of several surrogate models in optimizing a turbomachinery blade.

In this paper, the assessment of the performances of different surrogate models, that is, RSA, RBNN, and KRG models as well as a PRESS-based-averaged (PBA) model (a weighted average model), is performed for optimization of a dimpled channel considering two conflicting objectives related to the Nusselt number and the friction factor.

II. Problem Description and Numerical Procedure

A staggered dimpled channel shown in Fig. 1 is considered to evaluate the surrogate models. The optimization is performed at the Reynolds number based on a hydraulic diameter of 45,000. A Latin hypercube sampling (LHS) [12] design is used to generate design points in design space. The computational domain contains four half-dimples shown as the shaded region in Fig. 1.

In this work, three-dimensional Reynolds-averaged Navier–Stokes (RANS) analyses of fluid flow and convective heat transfer have been performed using commercial software CFX 5.7 [13] which employs an unstructured grid system. Modifications of the source terms in streamwise momentum and energy equations have been done to adopt the periodic boundary conditions to calibrate the gradual temperature and pressure, respectively, as described by Kim and Choi [2]. The shear stress transport (SST) turbulence model with automatic wall treatment [14] (no damping function in the near wall region) is used as a turbulence closure. The numerical model of Lai and So [15] is adopted for the modeling of turbulent heat flux.

Periodic boundary conditions on the surfaces normal to the streamwise direction, and also with symmetric conditions on the surfaces normal to the cross-streamwise direction, are applied for computations. An unstructured tetrahedral grid system is used with the hexahedral at the wall region to resolve the high-velocity gradient. In the present calculation, uniform heat flux is specified on the dimpled surface as shown in Fig. 1b.

III. Design Variables and Objective Functions

Figure 1 shows the definition of geometric variables d as dimple depth, D as the dimple print diameter, Pi is the pitch of the dimple, S the distance between dimples, and H the channel height. From these variables, nondimensional numbers H/D, d/D, D/S, and S/Pi can be found. However, in this work, S/Pi is set to 1.0 to reduce the number of variables. Hence, three design variables H/D, d/D, and D/S are selected for each of the optimizations.

To maximize the performance of the cooling systems, two objective functions F_{Nu} and F_f are selected. The Nusselt number based objective function which is responsible for enhancement of the heat transfer rate is defined as

$$F_{Nu} = 1/Nu_a \tag{1}$$

where

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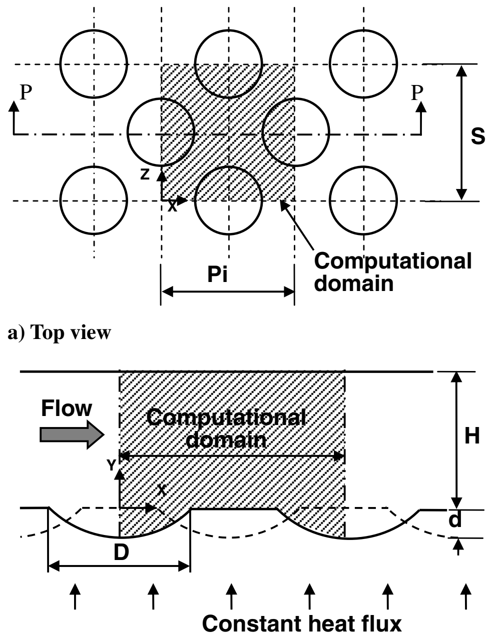
*Graduate Student, Department of Mechanical Engineering.

†Professor, Department of Mechanical Engineering; kykim@inha.ac.kr. Member AIAA (Corresponding Author).

‡Graduate Student, Department of Mechanical and Aerospace Engineering. Student Member AIAA.

§Professor, Department of Mechanical and Aerospace Engineering. Fellow AIAA.



a) Top view

b) View at cutting plane P-P

Fig. 1 Geometry and design variables.

$$Nu_a = \frac{\int_{A_d} Nu/Nu_s \, \mathrm{d}A}{A_d}$$

Here Nu and Nu_a are local and average Nusselt numbers, respectively. And Nu_s is the Nusselt number for the fully developed turbulent flows in a smooth pipe. A_d is the area of the heat transfer surface.

On the other hand, a pressure loss related objective function is defined as

$$F_f = \left(\frac{f}{f_0}\right)^{1/3} \tag{2}$$

where f_0 is a friction factor for fully developed flow in a smooth pipe, and f is defined as

$$f = \frac{\Delta p D_h}{2\rho U_b^2 Pi}$$

Δp, D_h, ρ, U_b, and Pi are pressure drop, channel hydraulic diameter, fluid density, average axial velocity, and dimple pitch, respectively.

Hence, the main motive of the optimizations is to minimize the objective functions F_{Nu} and F_f. To implement surrogate-based optimization procedures, the weighted-sum approach of the multi-objective optimization is applied to transform the two-objective problem into a single objective problem, and a surrogate approach is introduced.

IV. Optimization Methodology

Because the present problems have two objectives, F_{Nu} and F_f, a multi-objective optimization procedure has been employed before implementing the surrogate-based methodology. Initially, the variables are selected, and the design space is decided for improvement of system performance. Using the design of the experiment (DOE), the design points are selected, and at these design points the objective functions are calculated using a flow solver. Optimization of the objective function at these design points is carried out by surrogate-based approximation analysis.

A. Multi-Objective Optimization

The weighted-sum-of-objective-functions method [16], which is also known as the "naïve approach" to multi-objective optimization, is generally used for multi-objective optimization problems to make a mono-objective optimization problem. This method is frequently used in heat transfer problems [2–4]. In present optimizations, objectives F_{Nu} and F_f are linearly combined with a weighting factor w_f to constitute a mono-objective F by the naïve approach and the final objective (F) is defined by

$$F = F_{Nu} + w_f F_f \tag{3}$$

The weighting factor w_f is selected as per design requirement. In general, it is the "designer's choice."

B. Surrogate-Based Analysis

The next steps are to construct the surrogates using the formulated mono-objective problems and to find optimal points. The RSA, KRG, and RBNN models and a weighted average surrogate model PBA used in this work are described below.

The RSA method [17], a second-order polynomial function, is fitted to get the response surface in this work. RBNN [18] is a two-layer network that consists of a hidden layer of a radial basis function and a linear output layer. The parameters for fitting this surrogate model are spread constant (SC) and a user defined error goal (EG). The allowable error goal is decided from the allowable error from the mean input responses. In MATLAB [19], *newrb* is the function for RBNN network design.

The Kriging model [20] is an interpolating metamodeling technique that employs a trend model $f(\boldsymbol{x})$ to capture large-scale variations and a systematic departure $Z(\boldsymbol{x})$ to capture small-scale variations. Kriging postulation is the combination of a global model and departures of the following form:

$$\hat{F}(x) = f(x) + Z(x) \tag{4}$$

where xs are the design variables, $\hat{F}(x)$ represents the unknown function, $f(x)$ is a global model, and $Z(x)$ represents the localized deviations. $Z(x)$ is the realization of a stochastic process with mean zero and nonzero covariance. A linear polynomial function is used as a trend model and the systematic departure terms follow a Gaussian correlation function.

A weighted average model proposed by Goel et al. [9] is adopted in the present investigation. It is based on the PBA model (termed WTA3 by Goel et al. [9]). The predicted response is defined as follows for the PBA model:

$$\hat{F}_{\mathrm{wt.avg}}(x) = \sum_{i}^{N_{\mathrm{SM}}} w_i(x)\hat{F}_i(x) \tag{5}$$

where N_{SM} is the number of basic surrogate models used to construct the weighted average model. The ith surrogate model at design point $\boldsymbol{x}$ produces weight $w_i(\mathbf{x})$, and $\hat{F}_i(\mathbf{x})$ is the predicted response by the ith surrogate model.

Weights are decided using the guideline that the weights should reflect our confidence in the surrogate model such that the surrogate which produces high error has low weight, and thus a low contribution to the final weighted average surrogate, and vice versa. In this work, global weights are selected using a generalized mean square cross-validation error (GMSE) or PRESS (in RSA terminology) that is a global data-based measure of goodness (Goel et al. [9]).

Next the RSA, Kriging, and RBNN surrogate models are constructed using objective function values at design points. The constructed surrogates are used to search for optimal points using sequential quadratic programming (SQP) (function, *fmincon* in MATLAB [19]). Because the SQP is dependent on the initial guess of the optimal point, a series of trials have been performed before getting the final optimal point from any surrogate.

V. Results and Discussion

A staggered, dimpled channel is optimized for turbulent heat transfer enhancement with three design variables, H/D, d/D, and

D/S and two objective functions, F_{Nu} and F_f. The design space which is constituted by the lower and upper bounds of variables is determined as $0.2 < H/D < 1.0$, $0.1 < d/D < 0.3$, and $0.4 < D/S < 0.68$. Latin hypercube sampling [12], which is a space filling design, that is, the design points are equally spaced in design space, is used to find the design points. The surrogates are constructed and optimal points are searched after converting this biobjective (F_{Nu} and F_f) problem into a mono-objective (F) optimization problem by the weighted-sum approach. The weighting factor (w_f) assumed is 0.09.

Table 1 shows the errors (E_{cv}) coming from cross validation and the weights calculated for each surrogate to construct the PBA model. The least E_{cv} is produced by RSA and the highest is by RBNN model. Thus, the highest and the least weights are assigned to RSA and RBNN surrogates, respectively, to construct the PBA model.

Table 2 shows optimal points and objective function values ($F_{\mathrm{surrogate}}$) predicted by the surrogates. At surrogate predicted optimal points, the objective functions are computed by a RANS analysis to give F_{RANS}. The reference design shows the higher objective function value ($F_{\mathrm{reference}} = 0.717$) as compared to those obtained by the surrogates. On the average, about 23% improvement in the objective function value is achieved by the optimizations. The highest improvement ($F_{\mathrm{reference}} - F_{\mathrm{RANS}}$), 27.9% in the objective function value, is obtained from the KRG model, and the lowest, 19.1% from the RBNN model. Although RSA shows the least E_{cv} in Table 1, it does not produce the least $F_{\mathrm{surrogate}}$ or F_{RANS}, in Table 2. KRG gives the best F_{RANS} which is the main goal of the present optimization. KRG also shows the best accuracy of the surrogate prediction ($F_{\mathrm{RANS}} - F_{\mathrm{surrogate}}$) at the optimum point. RBNN predicts the best $F_{\mathrm{surroagte}}$, and at the same time the worst F_{RANS}, which indicates the worst accuracy of the surrogate at the optimum point. On the other hand, the multiple surrogate model, PBA, neither produces the best F_{RANS} nor the best $F_{\mathrm{surrogate}}$, but does not produce the worst one.

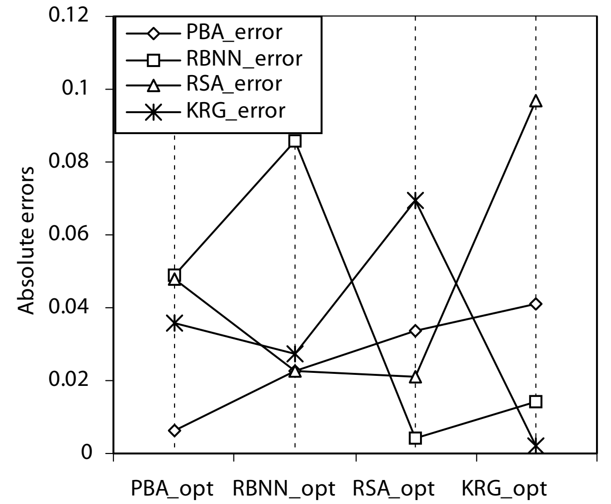


Fig. 2 Absolute errors, $|F_{\mathrm{RANS}} - F_{\mathrm{surrogate}}|$ in prediction by different surrogates at optimal points.

Table 3 shows the comparison between the four surrogate models, including the average error (over the four optima). And, Fig. 2 shows the absolute errors ($|F_{\mathrm{RANS}} - F_{\mathrm{surrogate}}|$) in predictions of surrogates at each optimal point. It is seen that PBA had the least error, and that even though Kriging obtained the best design, it was somewhat by chance. Although its average error was comparable to the other model, it happened to have a particularly small error at the optimum. This indicates that it is best to use all surrogates. It can be noticed that the extra cost of using all surrogates here was three additional simulations at optimal points over the use of a single surrogate.

In the application of surrogate models to turbomachinery blade optimization, Samad et al. [11] reported that the most accurate surrogate is the RSA model for the three different objective function tested. The RBNN model predicts the optimal points where the RANS computed objective function values show the best results for two objective functions and RSA produced the best result for one objective function. However, the KRG model shows the worst performance in all cases. The PBA model shows moderate

Table 1 Weights to construct weighted average model, PBA

Model	Cross-validation error, E_{CV}	Weight
RBNN	1.02E − 01	0.255
RSA	6.45E − 02	0.394
KRG	7.30E − 02	0.351

Table 2 Optimal designs suggested by various surrogates and corresponding RANS results

Optimal design variables		Surrogates: PBA	RBNN	RSA	KRG
H/D		0.243	0.201	0.200	0.284
d/D		0.248	0.248	0.256	0.248
D/S		0.621	0.622	0.496	0.621
Objective function statistics	$F_{\mathrm{surrogate}}$	0.555	0.495	0.576	0.515
	F_{RANS}	0.561	0.581	0.555	0.517
	$F_{\mathrm{RANS}} - F_{\mathrm{surrogate}}$	0.62E − 2	8.6E − 2	−2.1E − 2	0.19E − 2
	$F_{\mathrm{reference}}$ (RANS)	0.717			
	$F_{\mathrm{reference}} - F_{\mathrm{RANS}}$	0.157	0.137	0.162	0.201
	Reduction of F (%)	21.8	19.1	22.6	27.9

Table 3 Predictions by the other models at each optimal point

Objective function values predicted by	Optimal points predicted by: PBA	RBNN	RSA	KRG	Average $\lvert F_{\mathrm{RANS}} - F_{\mathrm{surrogate}} \rvert$
PBA	**0.555**	0.558	0.589	0.558	0.026
RBNN	0.512	**0.495**	0.560	0.531	0.039
RSA	0.609	0.603	**0.576**	0.614	0.047
KRG	0.525	0.553	0.625	**0.515**	0.034
RANS analysis	0.561	0.581	0.555	0.517	——

performance. The KRG model which shows the worst performance in a turbomachinery application shows the best performance in the present application to a heat transfer optimization. This reflects a strong problem-dependent nature of the surrogate models.

The multiple surrogate model, PBA, predicts neither the best optimal point, nor the worst. And this model shows the best reliability in predicting the objective function values in design space. The beauty of the PBA model is that it protects the designers to predict badly from poor performing surrogates, since this model is based on the global data-based error and contribution of the worst performing surrogate is the least to construct this surrogate.

VI. Conclusions

Optimization of a staggered dimpled channel has been carried out to evaluate the performances of surrogate models. An objective function is selected to enhance the heat transfer rate compromising with pressure loss. RSA, KRG, and RBNN models are taken as basic surrogates, and a weighted average surrogate model, PBA is formulated from the basic surrogates. As a result, the KRG model shows the best performance in predicting the optimum objective function value. But, in predicting the objective function in the design space, the multiple surrogate model, PBA, shows the highest accuracy. It is noted, however, that the KRG model shows the lowest error especially at its own optimal point. PBA predicts neither the best optimal point, nor the worst. Therefore, this weighted average model shows robustness in prediction as well as reliability.

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